ABSTract

The relation between time and space is difficult to describe because we can’t measure time directly and can only make inferences based on how the space between objects changes from one moment to the next. The widely accepted FLRW metric describes a complex expansion history where the entire universe jerks between deceleration and acceleration. Here we show that space instead grows constantly and quadratically with time and that – as a result of a tiny but ubiquitous acceleration produced at every point in space – the least understood aspects of our universe – the accelerated expansion of space, the high velocity of stars and galaxies around a common center, and the relation between mass and angular velocity in spiral galaxies – have a simple and geometric explanation.

*Keywords:* cosmology: large-scale structure of universe – cosmology: theory – galaxies: kinematics and dynamics

# Introduction

The known laws of motion and cosmology fail when applied to structures larger than our solar system. Stars move too quickly around galaxies to be bound by Newton’s laws of motion. The Virial Theorem tells us that galaxy clusters have too much kinetic energy for the visible mass they contain. And the light from distant supernovae is too dim to be understood in terms of Hubble’s law.

Most of our universe is missing when seen in the context of popular theories. Only a small fraction – less than 5% – is occupied by things that we can measure directly. The rest is hidden. Placeholders for some imagined forms of energy and matter are used in an attempt to reconcile the observed universe with ΛCDM predictions. This paper seeks to explain the missing universe not with placeholders, but with a new set of dynamical laws based on a hypothesis of constant and quadratic expansion of space with time.

# Time and Space

The redshifts of distant novae indicate that the expansion of our universe is accelerating (Riess, Filippenko et al. 1998)(Riess et al. 1998). The FLRW metric describes the kinematics of our universe as a series of series of jerks where the universe spontaneously inflates, accelerates, decelerates and accelerates again under a confederation of different, poorly understood forces. Alternatively, out of a desire for kinematic simplicity, we can hypothesize an expansion that has always been accelerating at a constant rate. This requires a new metric that naturally accumulates space with time.

We hypothesize a universe where the dimension of space is simply a function of the square of time and can be expressed with the formula:

|  |  |  |
| --- | --- | --- |
|  |  |  |

Where is the length of the universe from end to end in units of time-squared () and is the age of the universe (). Units of square seconds aren’t useful since distances are measured in units of space, so a way is needed to convert square time into something that can be observed directly:

|  |  |  |
| --- | --- | --- |
|  |  |  |

Where is, again, the length of the universe from end to end but in units of space () and is the ratio of one kilometer to a square second ()[[1]](#footnote-1). We’ll note that the second derivative of space with respect to time is constant and is:

This is the intrinsic acceleration of space at every point in our hypothetical universe. Put any two test particles together and the space between them will increase at this rate. We can now describe the geometry of our hypothetical universe with a suitable metric:

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

Where is an infinitiesmal distance between two events, is the velocity of light, is the proper time, is the age of the universe in the observer’s time, is the distance between the two points in space and is the polar coordinate, .

# The Rate of Expansion

Because of the uniform mass of the progenitor star, Type Ia Supernovae (SNe Ia) can be used as standard candles for measuring distances on a cosmological scale. The redshift is relatively easy to obtain and tells us how the universe has expanded since the light was emitted. The luminosity is less easy to measure accurately but provides a proxy for a distance measurement. The formula used to predict the luminous distance, from the redshift, using the formula in equation (1) is provided in Appendix A. Using equation (A3) and data from a large collection of SNe Ia, we can solve for the unknowns: the age of the universe and the acceleration constant.

Using the combined data from (Conley et al. 2011), (Rodney et al. 2012), (Jones et al. 2013), and (Rodney et al. 2015) and the parameters from (Rodney, et al. 2015) to normalize the sets, *Χ*2 curve fitting produces a near perfect match to the hypothesis: the age of the universe is 210 *Gyr* and the acceleration constant, is. *The reduced Χ2 on the Quadratically Expanding Space (QES) model of 0.72 is a better fit to the SNe Ia data than the ΛCDM model with a reduced Χ2 of 0.87 (using H0=67.3, ΩΛ =0.68, Ωm = 0.32) and with fewer fitting parameters and no assumption of exotic energy.* The difference between the models becomes more obvious beyond z ≈ 1.5 as the QES geometry predicts increasingly brighter SNe than ΛCDM as shown in Figure 4.

Figure 4. Observed distance moduli (circles) are plotted with the QES predictions (red) and the ΛCDM predictions (blue). The formula used for the distance moduli is 5log(𝑑L)+25 (𝑑L in units of Mpc).

# The Laws of Motion

The laws of motion will be different in a universe of quadratically expanding space than the laws in the linear universe described by Newton.

## The First Law of Motion

Objects at rest tend to accelerate and objects in motion tend to accelerate. All objects accelerate with the expansion of the universe unless acted on by an unbalancing force.

The space between any two objects increases at a rate of . Two objects can remain in uniform motion with each other only when there is an unbalancing force maintaining that uniform motion. In the absence of such an influence, the two bodies will drift apart.

## The Second Law of Motion

The change of momentum of a body is proportional to the unbalancing impulse impressed on the body plus the constant impulse impressed on the body by the expansion of the universe, and happens along the straight line on which that unbalancing impulse is impressed.

For a constant mass system at non-relativistic velocities, this statement can be expressed in equation form:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  | (2) |

Where is the initial velocity, is the acceleration due to the imbalance of forces and is the acceleration of expansion and is directed radially outward from every point in space in all directions.

## The Third Law of Motion

To every action, there is always an opposite and unequal reaction: The mutual actions of two bodies upon each other are always unequal and differ by the impulse provided by the expansion of the universe.

The third law tells us that the amount of momentum in any action between two bodies in quadratically expanding space is never completely conserved. When two forces oppose each other, the mutual force will be mitigated by the expansion of space which pulls all things apart.

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  | (3) |

An observer will measure an object moving away at a net rate of . That observer can turn around and look in the opposite direction and – with a powerful enough telescope – measure the same object moving away at a net rate of . The sum of the two net accelerations will equal to . In fact, every object in the universe will be seen to be moving away from itself at . The momentum of the object in the clockwise direction is equal to the momentum in the counter clockwise direction when the rate of expansion is included in the impulse.

If the rate of expansion is so tiny compared to the acceleration caused by other forces that – for all practical purposes – is zero, then those forces will appear to be equal and opposite and momentum will appear to be conserved.

# Spiral Galaxies

The effects of quadratic expansion would be measurable in low acceleration environments where aF ≈ a0. Spiral galaxies provide such a domain. We can predict the velocity of an object in a circular orbit knowing the enclosed mass and the distance from the center in quadratically expanding space by applying the force of gravity and the acceleration of centripetal motion to equation (2) to get:

|  |  |  |
| --- | --- | --- |
|  |  | (4) |
|  |  | (5) |
|  |  | (6) |

Where, for a given radius, *r, M(r)* is the enclosed mass, *v* is the angular velocity and *G* is the gravitational constant. A mass model of a spiral galaxy is required to compare the prediction of equation (6) against the ΛCDM predictions. For QES, the mass model consists of a bulge and a disk; a halo of imaginary matter is added for ΛCDM. The bulge mass in both models is assumed to have a de Vaucouleurs profile (de Vaucouleurs 1958):

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

Where is the mass of the bulge, is 7.6695 and is the density at the scale radius, . Because an exact solution is computationally expensive and approximations are inaccurate, a deprojection table from (Young 1976) was used in the search for the best-fit parameters for the bulge mass.

The mass for an exponentially thin disk, , for both models is calculated from (Freeman 1970) as:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  | (8) |

where is the central surface density, is the scale length of the disk.

The NFW profile (Navarro 1996) is a good model for the mass of the imaginary halo, , in the outer regions of galaxies. Integrating the density profile, the mass can be expressed as:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  | (9) |

where is the characteristic density and is the scale radius of the halo.

|  |  |  |
| --- | --- | --- |
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Figure 5. A sampling of the angular velocity predictions for QES (red) and ΛCDM (blue) and the observed data (gray). All horizontal axes are radius in kpc. Vertical axes are angular velocity in km s-1.

Both models are fitted to the data from (Sofue, Honma, & Omodaka 2009) for the Milky Way Galaxy, (Chemin, Carignan, & Foster 2009) for Messier 31, (Sofue et al. 1999), (Sofue et al. 2003), (Sofue 2016), (Garrido et al. 2005), (Noordermeer et al. 2007) and (Martinsson et al. 2013). The best-fit ΛCDM model parameters are taken from (Sofue 2016). A sample of the resulting rotational curves can be found in Figure 5.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | ΛCDM | | | | | | | QES | | | | |
| Name | ab | Mb | ad | Md | *h* | Mh | *Χ2* | ab | Mb | ad | Md | *Χ2* |
|  | (kpc) | (1010 M⊙) | (kpc) | (1010 M⊙) | (kpc) | (1010 M⊙) |  | (kpc) | (1010 M⊙) | (kpc) | (1010 M⊙) |  |
| Milky Way | 0.52 | 1.65 | 3.19 | 3.41 | 12.50 | 5.02 | 10.92 | 0.41 | 1.13 | 3.59 | 7.51 | 4.03 |
| Messier 31 | 1.30 | 3.36 | 4.30 | 7.70 | 30.50 | 27.90 | 27.85 | 1.00 | 0.00 | 3.78 | 16.64 | 19.23 |
| NGC 0253 | 0.93 | 1.60 | 1.90 | 1.90 | 7.90 | 3.50 | 326.48 | 0.64 | 0.86 | 2.46 | 5.93 | 155.70 |
| NGC 0266 | 2.89 | 2.12 | 6.60 | 25.10 | 87.10 | 239.30 | 1,545.58 | 10.00 | 5.46 | 11.96 | 79.72 | 904.21 |
| NGC 0342 | 0.64 | 0.52 | 1.60 | 1.60 | 12.00 | 7.90 | 91.78 | 1.30 | 0.83 | 2.85 | 5.83 | 13.92 |
| NGC 0598 | 7.66 | 0.22 | 6.10 | 0.50 | 8.50 | 2.40 | 29.69 | 1.00 | 0.00 | 1.92 | 0.66 | 26.74 |
| NGC 0660 | 0.57 | 0.94 | 0.60 | 0.20 | 9.20 | 3.80 | 25.34 | 0.39 | 0.59 | 1.25 | 1.53 | 135.74 |
| NGC 0891 | 0.71 | 1.84 | 3.10 | 3.10 | 6.40 | 3.10 | 313.24 | 0.64 | 1.29 | 2.82 | 7.11 | 276.07 |
| NGC 1365 | 0.90 | 2.51 | 3.30 | 6.60 | 14.40 | 10.40 | 439.67 | 0.77 | 1.60 | 3.68 | 14.47 | 195.60 |
| NGC 1642 | 3.06 | 2.78 | 3.00 | 6.00 | 88.90 | 83.10 | 260.34 | 1.04 | 0.06 | 2.34 | 7.87 | 172.40 |
| NGC 1808 | 0.66 | 1.38 | 3.00 | 3.20 | 2.00 | 0.40 | 227.54 | 0.75 | 1.25 | 1.99 | 3.16 | 145.64 |
| NGC 2403 | 0.14 | 0.02 | 0.20 | 0.00 | 7.60 | 2.90 | 42.32 | 0.60 | 0.08 | 2.20 | 1.21 | 56.67 |
| NGC 2543 | 0.99 | 0.15 | 3.80 | 4.00 | 20.60 | 14.10 | 337.91 | 3.69 | 0.38 | 6.67 | 13.93 | 180.47 |
| NGC 2599 | 1.08 | 10.03 | 3.50 | 12.50 | 50.20 | 43.80 | 1,314.98 | 2.96 | 24.62 | 3.72 | 2.24 | 975.53 |
| NGC 2649 | 1.56 | 0.16 | 3.10 | 3.10 | 19.70 | 11.70 | 124.38 | 9.14 | 0.21 | 4.07 | 7.23 | 9.39 |
| NGC 2654 | 1.07 | 1.45 | 2.80 | 5.70 | 52.90 | 38.20 | 346.30 | 0.45 | 0.37 | 2.49 | 7.39 | 185.05 |
| NGC 2903 | 2.50 | 5.24 | 3.80 | 8.00 | 7.60 | 5.00 | 670.73 | 5.20 | 10.11 | 4.38 | 13.41 | 366.80 |
| NGC 2985 | 0.56 | 0.39 | 1.10 | 1.10 | 9.70 | 4.20 | 25.14 | 0.67 | 0.49 | 1.51 | 2.18 | 636.57 |
| NGC 3079 | 0.69 | 2.63 | 3.80 | 4.40 | 16.20 | 9.50 | 477.67 | 0.74 | 2.25 | 4.36 | 10.03 | 233.76 |
| NGC 3198 | 6.21 | 0.50 | 3.10 | 1.60 | 13.40 | 6.20 | 165.51 | 1.00 | 0.00 | 3.42 | 3.85 | 282.15 |
| NGC 3521 | 0.72 | 1.52 | 1.60 | 3.50 | 22.60 | 14.00 | 406.18 | 0.07 | 0.18 | 1.58 | 6.72 | 73.55 |
| NGC 3628 | 0.84 | 1.52 | 3.60 | 3.60 | 8.50 | 4.80 | 170.05 | 1.32 | 2.00 | 5.07 | 11.04 | 34.08 |
| NGC 3900 | 1.66 | 2.44 | 6.30 | 11.90 | 7.70 | 2.80 | 252.55 | 1.63 | 1.77 | 4.49 | 11.34 | 42.47 |
| NGC 3982 | 0.55 | 0.09 | 1.20 | 1.20 | 23.60 | 13.80 | 102.88 | 3.90 | 0.40 | 1.67 | 2.37 | 8.92 |
| NGC 4258 | 0.47 | 1.19 | 0.80 | 0.90 | 19.40 | 16.10 | 209.35 | 1.76 | 4.18 | 8.24 | 5.05 | 111.67 |
| NGC 4303 | 0.33 | 0.15 | 2.10 | 1.10 | 11.90 | 4.50 | 318.52 | 0.26 | 0.11 | 2.16 | 2.04 | 429.51 |
| NGC 4321 | 1.32 | 2.76 | 7.60 | 16.80 | 7.10 | 4.20 | 347.82 | 1.73 | 2.96 | 6.80 | 23.93 | 91.52 |
| NGC 4527 | 0.41 | 0.94 | 2.60 | 1.70 | 12.20 | 7.40 | 502.75 | 0.45 | 0.95 | 4.29 | 7.66 | 423.65 |
| NGC 4565 | 3.05 | 6.40 | 4.00 | 4.60 | 19.20 | 16.30 | 551.64 | 10.69 | 26.75 | 5.48 | 1.62 | 268.58 |
| NGC 4569 | 1.39 | 0.66 | 12.00 | 39.70 | 9.80 | 3.50 | 892.04 | 2.52 | 1.01 | 20.00 | 89.42 | 462.48 |
| NGC 4736 | 0.82 | 1.09 | 0.90 | 0.80 | 7.30 | 2.30 | 212.82 | 2.00 | 2.82 | 1.49 | 0.78 | 138.75 |
| NGC 4945 | 0.36 | 0.69 | 0.30 | 0.30 | 9.00 | 5.60 | 92.88 | 11.10 | 9.95 | 0.15 | 0.48 | 95.20 |
| NGC 5033 | 1.04 | 3.76 | 5.70 | 11.40 | 38.90 | 41.20 | 421.40 | 0.69 | 2.03 | 5.59 | 21.56 | 90.44 |
| NGC 5055 | 2.96 | 4.15 | 1.80 | 1.70 | 7.70 | 4.20 | 385.20 | 2.63 | 1.87 | 2.12 | 5.15 | 277.36 |
| NGC 5236 | 0.19 | 0.47 | 3.00 | 1.80 | 8.10 | 4.40 | 249.21 | 0.16 | 0.40 | 2.88 | 4.83 | 618.65 |
| NGC 5457 | 2.76 | 2.90 | 2.40 | 2.20 | 8.10 | 4.60 | 426.31 | 14.14 | 25.91 | 1.00 | 0.00 | 210.65 |
| NGC 5907 | 1.60 | 2.76 | 6.90 | 13.80 | 6.70 | 3.40 | 402.47 | 1.75 | 2.13 | 5.30 | 17.68 | 96.58 |
| NGC 6946 | 0.36 | 0.92 | 3.80 | 4.20 | 9.40 | 5.60 | 199.42 | 0.36 | 0.81 | 4.47 | 11.20 | 112.59 |
| NGC 7013 | 0.82 | 0.64 | 0.80 | 0.80 | 11.80 | 5.70 | 229.69 | 2.23 | 1.34 | 1.18 | 1.74 | 170.75 |
| UGC 03993 | 4.35 | 16.59 | 4.80 | 7.50 | 135.10 | 214.90 | 245.45 | 1.78 | 4.36 | 3.70 | 16.39 | 124.69 |
| Average |  |  |  |  |  |  | 335.55 |  |  |  |  | 221.44 |

Table 1 – Comparing the ΛCDM predictions against the QES predictions.

The results of comparing the best-fit parameters of the two models against the observed data are collected in Table 1. Out of the 43 galaxies selected for the (Sofue 2016) study, three galaxies (NGC 5533, UGC 02916, UGC 11852) were rejected because the velocities of gas in the outer disks were incompatible with regular motion. Except for the Milky Way and Messier 31, the errors for the observed data were assumed to be unity (1 km s-1) and the modified *Χ2* method described in (Sofue 2016) was used (which is, essentially, a least-squares method). Of the 40 galaxies, the observed data in 33 were significantly better matches to the QES predictions than the ΛCDM predictions. In aggregate, the QES model is also a better match with a average modified *Χ2* of 211.44 compared to ΛCDM with a average modified *Χ*2 of 335.55. *Note that ΛCDM requires imaginary matter, two additional parameters and still doesn’t fit the data as well as QES.*

# Missing Matter

Quadratically expanding space would have the appearance of missing matter to an observer who believed that the expansion of space with time had no impact on the laws of motion. That is, the application of Newton’s Second Law of Motion to a dynamic situation in the QES universe will result in the appearance of more mass than can be accounted for by the matter present.

Figure 6. The path of a baryonic test particle in QES space (red) and in FLRW space (blue). Horizontal and vertical axes are in kpc.

The trajectory of a test particle around a 109 M⊙ mass is shown in Figure 6 for both the QES and FLRW space. This particle has an initial position of (20, -20) kpc and a velocity of 200 km s-1 in the vertical axis. Based on the path in QES space, an observer using Newtonian dynamics would incorrectly measure a much larger mass than actually exists.

In order to quantify this effect, we examine the balance between gas pressure and gravity in galaxy clusters to test the theory that the missing matter of Newtonian physics is the simply the local effects of an expanding universe. These clusters are believed to be in a state of hydrostatic equilibrium where the motion caused by the pressure of the intracluster medium (ICM) as it heats is offset by the force of gravity. Using the formulas from (Sarazin 1988) for the acceleration resulting from the expanding gas and equation (2), the sum of forces in the QES universe can be written as:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  | (10) |

where is the mass enclosed inside a radius, r, in QES space, is the gas density, and is the pressure of the gas. For the same system, Newton would measure a larger mass:

|  |  |  |
| --- | --- | --- |
|  |  | (11) |

where is the mass enclosed inside a radius, r, in FLRW space. Therefore, an observer using Newton’s Second Law of Motion – in a universe where the space between objects increases at a rate of – would be unable to account for the following mass:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  | (12) |

We examined the gas and Dark Matter components of galaxy clusters in (Vikhlinin et al. 2006), (Gastaldello et al. 2007), (Gonzalez et al. 2013), and (Eckert et al. 2016). Once we normalized the data sets using a Hubble Constant of 67.3, the Dark Matter mass is simply the difference of the total mass and the gas fraction. We then compared the Dark Matter mass against the missing mass of equation (12) for 219 galaxy clusters and the results are displayed in Figure 7. *A coefficient of determination, R2, of 0.88 strongly suggests that Dark Matter is the error one would calculate using Newtonian physics in quadratically expanding space.*

Figure 7. The Dark Matter mass, with H0 = 67.3, of Vikhlinin (red), Gastaldello (green), Gonzalez (purple), and Eckert (aqua) overlaid with the function (black).

# The Fundamental Plane

The Tully-Fisher Relationship is disproof of Newton’s Second Law of Motion. Newton predicts that a full spectrum of masses for spiral galaxies are allowed. Just increase the angular velocity and/or radius and a any mass (or luminosity) is allowed:

Yet that’s not what’s observed. Newton’s prediction has been falsified by a universe where the mass of a spiral galaxy can be no greater than the fourth power of the angular velocity and is completely independent of the radius:

The Second Law of Motion in quadratically expanding space defines a plane of possible masses given the two free parameters: the angular velocity and the radius of orbiting objects. Formula (5) can be rearranged to predict the mass given the velocity and radius.

|  |  |  |
| --- | --- | --- |
|  |  | (13) |



Figure 8. The Fundamental Plane of Mass. Radius is in km, velocity in km s-1 and the vertical axis is in M⊙. Solid line is the maximum mass allowed by the QES Second Law of Motion.

Figure 8 shows the plot of mass as a function of radius and angular velocity. The radius where the maximum mass will be found is:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  | (14) |

Substituting the equation (14) back into equation (13) yields the formula for the maximum mass given the velocity:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  | (15) |

Here the velocity is in units of km s-1 and the maximum mass, , is in units of M⊙. A study of the relation between velocity and mass was conducted in (McGaugh 2012) with the assumption that luminosity is an imperfect proxy for mass. Gas rich galaxies are a better proxy as they are not as effected by the vagaries of the stellar mass-to-light ratios. Those results are displayed in Figure 9 and overlaid with equation (15). *The reduced Χ2 of 0.65 demonstrates that the QES Second Law of Motion accurately predicts the Baryonic Tully-Fisher Relationship (BTFR).*

Figure 9. The relation between angular velocity and mass. Circles are the combined gas and stellar mass of the gas-rich galaxies and the solid line is the maximum mass allowed by the QES Second Law of Motion.

# The Problem with General Relativity

General Relativity (GR) is a very successful theory on the scale of a solar system. It accurately predicts the precession of the perihelion of Mercury and accurately predicts the deviation of stars against the gravitational influence of the sun. However, if fails on scales larger than this. It failed to predict the velocity curves of spiral galaxies, failed to predict the degree of light bending around galaxy clusters and failed to predict the observed temperature deviations in the Cosmic Microwave Background (CMB) radiation. Until now, the only way to get Einstein’s theory to agree with large-scale observations is to imagine a form of matter that has no analog in the real universe, has failed every attempt at detection and has no theoretical foundation that has passed a single experimental test.

The mathematical foundation of General Relativity is the Geodesic Equation which basically asks what hidden shape of spacetime would allow objects that appear to be accelerating (e.g. the Moon, the Earth, the Sun), to be in "free fall"; a state where no forces are acting on them. The Christoffel expression answers the 'what hidden shape' part of the question, but the “free fall” geodesic assumes the expected acceleration of an object in this state is zero. The problem with General Relativity, then, is the assumption of Newtonian Dynamics and is easily fixed with our hypothesis that objects at rest accelerate:

|  |  |  |
| --- | --- | --- |
|  |  | (16) |

A common criticism of the QES model is that it disagrees with General Relativity and this couldn’t be further from the truth as a theory of constant acceleration fixes the Theory of General Relativity. The Geodesic Equation now gives the proper shape of spacetime without employing imaginary matter.

# Conclusion

The universe did not start with a bang. It emerged slowly from the singularity at a constant rate and continues to grow today at the same rate. After 3 seconds, it was the size of an atom. After 1 day it was roughly the size of a grapefruit and it took a year to grow to 20 km. This acceleration constant appears at all scales of the universe, from small galaxies, to galaxy clusters, to distant supernovae, but most importantly, this acceleration is experienced locally and must be incorporated into the laws of motion.

Figure 10. The relation of space (Gly) to time (Gyr).

A complete comparison of cosmological models would include an analysis of Cosmic Microwave Background radiation, Baryonic Acoustic Oscillations and Big Bang Nucleosynthesis which provide constraints on the early universe. An exhaustive analysis is beyond the scope of this document but a back-of-the-envelope calculation using the QES geometry show the luminous distance to the Surface of Last Scattering (at z = 1090), *dsls*, is 69.72 Gpc using equation (A3). The age of the universe at decoupling was 6.37 Gyr using equation (A2) and so – assuming the speed of sound in plasma is – the sound horizon, *rs*, at decoupling is 1.13 Gpc. Using , the first acoustic peak at 194.21*l* in the QES model is consistent with the observed value of 220*l* from the 2013 Plank Study (Ade et al. 2014).

The QES model is a better fit to the observed data than ΛCDM and requires less assumptions. The universe can be modelled employing a metric that expands naturally at a constant rate of . The age of the QES universe is *210 Gyr* and it has a circumference of *90.35 Gly*. Figure 10 shows that a photon from Sasquatch (z of 1.39) has been travelling for 74.3 *Gyr* and has an effective velocity of 0.29c as it fights for progress against quadratically expanding space. The age of this universe is more than an order of magnitude older than that predicted by the ΛCDM model which requires a complex series of ‘phases’ where the entire universe jerks back and forth by a coalition of changing forces. In contrast, the QESgeometry employing a constant acceleration is a kinematically trivial solution to the SNe Ia data with no requirement for energies or particles or fields that have yet to be discovered.

Finally, a model of quadratically expanding space solves the mystery of the observed relationship between luminosity (mass) and angular velocity of spiral galaxies. The Tully-Fisher Relationship is incontrovertible evidence of the correct Second Law of Motion written across the night sky.

# Appendix A

Here is provided the derivation of the formulas used to predict the luminous distance moduli to an SNe Ia given the redshift. The distance travelled by a photon in a quadratically expanding universe, where the light is travelling towards the destination at the velocity of light, *c*, and the universe is effectively travelling away at the rate of expansion, a0, is described by the following integration:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  | (A1) |

Where is light travel distance, is the light travel time, is the age of the universe at the time of the observation and is the age of the universe at the time the photon was emitted.

The redshift is a direct indication of how a single wavelength of light has expanded from the time it was emitted to the time it was observed. It is also a direct measurement of how the size of the entire universe has grown during that time. From the redshift, we can calculate the light travel time:

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  | (A2) |

where is the redshift, is the size of the universe when the photon was emitted and is the size of the universe when it is observed.

Now, to calculate the luminous distance in terms of the light travel distance and light travel time, we make use of the relationship between the observed distance, , and the distance between the objects when the light was emitted, , which is:

|  |  |  |
| --- | --- | --- |
|  |  |  |

This simply states that the space between the observer and emitter as a proportion of the entire universe doesn’t change with time.

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

This represents the total change in distance that has occurred between the observer and the emitter while the light was travelling.

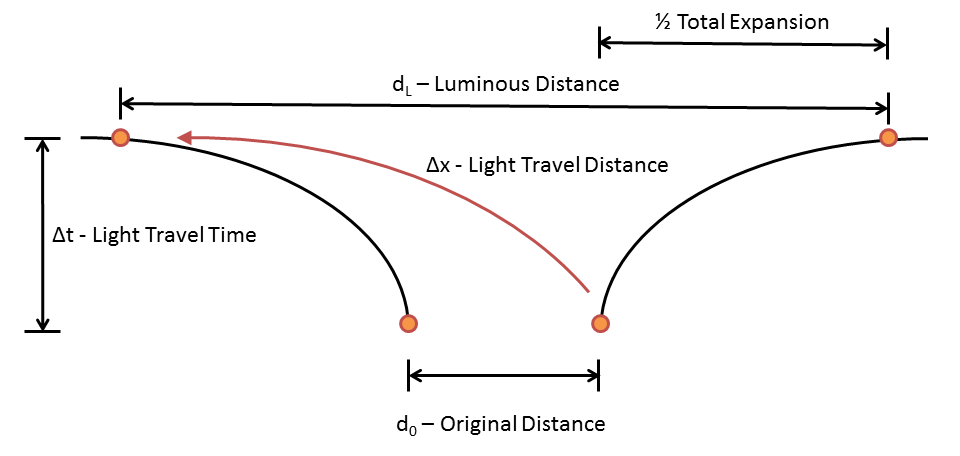


Figure 11. Total light travel distance, Δ𝑥, is the observed distance to the object, 𝑑L, less one half of the expansion that occurred while the light was travelling.

The next step exploits the fact that the light travel distance, is the difference between the luminous distance, and half of the total change in distance described in the previous formula as depicted in Figure 11.

|  |  |  |
| --- | --- | --- |
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Substituting equations (A1) and (A2) into this formula gives us the relation between the red-shift and the luminous distance:

|  |  |
| --- | --- |
|  | (A3) |

# References

Ade, P. A. R., et al. 2014, AstronAstrophys, 571, A16

Chemin, L., Carignan, C., & Foster, T. 2009, The Astrophysical Journal, 705, 1395

Conley, A., et al. 2011, AstrophysJSuppl, 192, 1

de Vaucouleurs, G. 1958, The Astrophysical Journal, 128, 465

Eckert, D., et al. 2016, Astronomy & Astrophysics, 592, A12

Ellis, G. F. R., & van Elst, H. 1999, NATO SciSerC, 541, 1

Freeman, K. C. 1970, The Astrophysical Journal, 160, 811

Garrido, O., Marcelin, M., Amram, P., Balkowski, C., Gach, J., & Boulesteix, J. 2005, Monthly Notices of the Royal Astronomical Society, 362, 127

Gastaldello, F., Buote, D. A., Humphrey, P. J., Zappacosta, L., Bullock, J. S., Brighenti, F., & Mathews, W. G. 2007, The Astrophysical Journal, 669, 158

Gonzalez, A. H., Sivanandam, S., Zabludoff, A. I., & Zaritsky, D. 2013, The Astrophysical Journal, 778, 14

Jones, D. O., et al. 2013, AstrophysJ, 768, 166

Martinsson, T. P., Verheijen, M. A., Westfall, K. B., Bershady, M. A., Andersen, D. R., & Swaters, R. A. 2013, Astronomy & Astrophysics, 557, A131

McGaugh, S. S. 2012, The Astronomical Journal, 143, 40

Navarro, J. F. 1996. in Symposium-international astronomical union, The structure of cold dark matter halos (Cambridge University Press), 255

Noordermeer, E., Van Der Hulst, J., Sancisi, R., Swaters, R., & Van Albada, T. 2007, Monthly Notices of the Royal Astronomical Society, 376, 1513

Riess, A. G., et al. 1998, The Astronomical Journal, 116, 1009

Rodney, S. A., et al. 2012, The Astrophysical Journal, 746, 5

Rodney, S. A., et al. 2015, AstronJ, 150, 156

Sarazin, C. L. 1988,

Seo, H.-J., & Eisenstein, D. J. 2005, The Astrophysical Journal, 633, 575

Sofue, Y. 2016, Publications of the Astronomical Society of Japan, 68

Sofue, Y., Honma, M., & Omodaka, T. 2009, Publications of the Astronomical Society of Japan, 61, 227

Sofue, Y., Koda, J., Nakanishi, H., & Onodera, S. 2003, Publications of the Astronomical Society of Japan, 55, 59

Sofue, Y., Tutui, Y., Honma, M., Tomita, A., Takamiya, T., Koda, J., & Takeda, Y. 1999, The Astrophysical Journal, 523, 136

Vikhlinin, A., Kravtsov, A., Forman, W., Jones, C., Markevitch, M., Murray, S., & Van Speybroeck, L. 2006, The Astrophysical Journal, 640, 691

Young, P. 1976, The Astronomical Journal, 81, 807

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1. The choice of units for the dimensions is arbitrary. [↑](#footnote-ref-1)